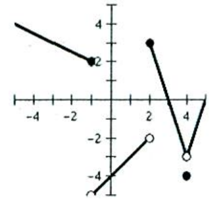
The graph of f is shown on the right. Evaluate the following limits. Write "DNE" if a limit does not exist. **SCORE: _____ / 3 PTS**



[a] $\lim\limits_{x\to 4} \dfrac{3x}{5-f(x)}$

$= \dfrac{\lim\limits_{x\to 4} 3x}{\lim\limits_{x\to 4} 5 - \lim\limits_{x\to 4} f(x)}$ (1)

$= \dfrac{12}{5--3} = \dfrac{12}{8} = \dfrac{3}{2}$ (1)

[b] $\lim\limits_{x\to -1^+} f(x)$

$= -5$ (1)

Prove that $\lim\limits_{x\to 0} x^6 \cos\dfrac{1}{x^3} = 0$. **SCORE: _____ / 3 PTS**

$-x^6 \le x^6 \cos\dfrac{1}{x^3} \le x^6$ (1)

$\lim\limits_{x\to 0} -x^6 = 0$ ($\frac{1}{2}$)

$\lim\limits_{x\to 0} x^6 = 0$ ($\frac{1}{2}$)

OK IF TOGETHER IN A COMPOUND EQUALITY

SO $\lim\limits_{x\to 0} x^6 \cos\dfrac{1}{x^3} = 0$ BY SQUEEZE THEOREM (1)

If $\lim\limits_{r\to -1} \dfrac{4+ar-r^6}{1+r}$ exists, find the value of a. **SCORE: _____ / 2 PTS**

SINCE $\lim\limits_{r\to -1}(1+r) = 0$, THE ORIGINAL LIMIT EXISTS ONLY IF $\lim\limits_{r\to -1}(4+ar-r^6) = 0$

OK IF SIMPLIFIED

IE. $4 - a - 1 = 0$ (1)

$a = 3$ (1)

Using complete sentences and proper mathematical notation, write the formal definition of "vertical asymptote". **SCORE: _____ / 2 PTS**

f HAS A VERTICAL ASYMPTOTE AT a IFF

$\lim_{x \to a^+} f(x) = \infty$ OR $\lim_{x \to a^+} f(x) = -\infty$ OR $\lim_{x \to a^-} f(x) = \infty$

GRADED BY ME

OR $\lim_{x \to a^-} f(x) = -\infty$

Evaluate the following limits. Write "DNE" if a limit does not exist. **SCORE: _____ / 7 PTS**

[a] $\lim_{y \to -4} \frac{y^2 + 2y - 8}{2y^2 + 5y - 12}$ $\frac{0}{0}$

$= \lim_{y \to -4} \frac{(y+4)(y-2)}{(y+4)(2y-3)}$

$= \boxed{\lim_{y \to -4} \frac{y-2}{2y-3}}$ (1)

$= \frac{-6}{-11} = \boxed{\frac{6}{11}}$ $\left(\frac{1}{2}\right)$

[b] $\lim_{b \to 3} \frac{b - \sqrt{b+6}}{6 - 2b}$ $\frac{0}{0}$

$= \boxed{\lim_{b \to 3} \frac{b^2 - b - 6}{(6-2b)(b + \sqrt{b+6})}}$ (1)

$= \lim_{b \to 3} \frac{(b-3)(b+2)}{-2(b-3)(b + \sqrt{b+6})}$

$= \boxed{\lim_{b \to 3} \frac{b+2}{-2(b + \sqrt{b+6})}}$ (1)

$= \frac{5}{(-2)6}$

$= \boxed{-\frac{5}{12}}$ $\left(\frac{1}{2}\right)$

[c] $\lim_{t \to -5} \frac{\frac{6}{t-1} - \frac{4}{t+3}}{t^2 + 25}$

$= \frac{\frac{6}{-6} - \frac{4}{-2}}{25 + 25}$

$= \frac{-1+2}{50}$

$= \boxed{\frac{1}{50}}$ (1)

[d] $\lim_{x \to -2} f(x)$ where $f(x) = \begin{cases} 2x+1, & \text{if } x < -2 \\ x-1, & \text{if } -2 < x < 3 \\ 5-x, & \text{if } x > 3 \end{cases}$

$\lim_{x \to -2^-} f(x) = \boxed{\lim_{x \to -2^-} (2x+1) = -3}$ $\left(\frac{1}{2}\right)$ REQUIRED

$\lim_{x \to -2^+} f(x) = \boxed{\lim_{x \to -2^+} (x-1) = -3}$ $\left(\frac{1}{2}\right)$ REQUIRED

SO $\boxed{\lim_{x \to -2} f(x) = -3}$ (1)